SIZE AND SHAPE OF PLOT IN RELATION TO FIELD EXPERIMENTS WITH SUGAR BEETS[1]

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INTRODUCTION

The literature dealing with field experiments is very extensive. Studies to determine the most efficient size and shape of plot and the value of replication have been conducted with a great variety of crops and under widely diverse conditions. The committee for the standardization of field experiments of the American Society of Agronomy has given a complete bibliography dealing with this subject.[3]

Studies on size and shape of plot in relation to field experiments with sugar beets (*Beta vulgaris* L.) are of special interest, since they must be concerned not only with yield but also with sugar percentage, total sugar per plot, and percentage of purity. The optimum size and shape of plot for determining one of these characters is not necessarily the most efficient for the other characters. All four must be considered and their relationship to one another determined.

Pritchard[4] studied the value of check plots and repeated plantings in variety trials with sugar beets and concluded that frequent checks could be used to advantage in calculating the error of the experiment. He also found that the error of the experiment was reduced with increased replication. The reduction was most pronounced as replication increased to 7; smaller gains were obtained for greater replications, i. e., up to 10.

The yields of relatively small plots, such as are used in agronomic experiments, will usually be determined by harvesting the entire plot. Except for very small plots it would not seem necessary to analyze all the beets in a plot for sugar percentage. Sampling methods must be resorted to, therefore, in selecting beets for sugar analysis. This introduces another error, the error in sampling. The writer has studied sampling technic with sugar beets in relation to the determination of sugar percentage of the roots and has discussed the problem in some detail.[5] The present study was made from data obtained from the same field as was used for the study of sampling technic. The data on sugar percentage[6] and apparent purity[7] used in this

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[3] AMERICAN SOCIETY OF AGRONOMY, COMMITTEE FOR THE STANDARDIZATION OF FIELD EXPERIMENTS. REPORTS * * * . Jour. Amer. Soc. Agron. 18: 1143-1144, 1926; 22: 1056-1061. 1930.

[4] PRITCHARD, F. J. THE USE OF CHECKS AND REPEATED PLANTINGS IN VARIETAL TESTS. Jour. Amer. Soc. Agron. 8: 65-81, illus. 1916.

[5] IMMER, F. R. A STUDY OF SAMPLING TECHNIC WITH SUGAR BEETS. Jour. Agr. Research 44: 633-647

[6] Sugar percentage (as used here) is the percentage of sucrose in the beet.

[7] Apparent purity (as used here) = $\frac{\text{Percentage polarization} \times 100}{\text{Corrected Brix spindle reading}}$.

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study were obtained by sampling methods but slightly different from those of the previous study. The analysis of the data will be made in the same manner as before.

MATERIAL AND METHODS

A small field of sugar beets of approximately nine-tenths of an acre in area, planted with Pioneer variety in 1930, was chosen for the experiment. The field had been cropped in a uniform manner for several years prior to 1930, and cultural conditions were uniform throughout the field during the growing season. The beets were planted in rows 22 inches apart on May 5. At thinning time the field was cross marked, dividing the rows into 12-inch units, and a single beet was left in each unit. While the spacing was not exactly 12 inches in all cases, slight adjustments being made between the cross marks, the number of beets left after thinning averaged very nearly one per foot of row. The plot was cultivated during the growing season with hand cultivators. At harvest time the field was marked out into plots 2 rods (33 feet) long, with 2-foot alleys between the ends of adjacent plots. The beets in these alleys were removed by hand before harvest in order to minimize errors due to the beets being dragged from one plot to another by the beet lifter. The field actually harvested, after removing border rows and the ends of the field, consisted of 60 rows 350 feet long, the rows being subdivided into 10 series of plots each 2 rods long, with 2-foot alleys between. All beets adjacent to noticeable skips in the row were removed before harvest. After correction for such skips the stand was approximately 85 per cent of a perfect stand. The beets were harvested during the first week in October.

The beets were lifted with a regular beet lifter. A sample of 10 beets was next taken from each ultimate unit (1 row 2 rods long) at uniform intervals over the entire length of the row. Approximately every third beet was taken for the sample, the exact number depending on the total number of beets in the plot after removal of beets adjacent to skips. These sample beets were topped, placed directly in labeled waterproof bags, and removed to the laboratory. There they were weighed, washed clean of dirt, and weighed again. The entire sample was then ground by a grinder of the multiple-saw type, the juice was extracted from the pulp by a hydraulic press under constant pressure, and about 1½ pints of juice was used for the determination of sugar percentage and apparent purity. The sugar percentage in the juice was determined[8] by Horne's dry lead method, and the apparent purity was determined as the ratio of sugar percentage to the corrected Brix reading.

The remaining beets in the plot were then topped and weighed. The combined weight of the beets taken for sugar samples and those from the remainder of the plot, corrected for tare, was considered the yield of the plot. The beets were counted at weighing time, and yields were calculated on the basis of 33 beets per plot of 1 row 2 rods long. The tare, as determined from the sample, was only about 5 per cent and fairly uniform from plot to plot. The harvest-

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[8] A conversion factor of 95.2 was used as a constant to convert the percentage of sucrose in the juice to percentage of sucrose in the beet.

ing operations were completed in 2½ days and the sugar percentage determinations in 3½ days.

The "analysis of variance" method devised by Fisher was used in analyzing the data.[9] The principle of the method may be given very briefly. If the total variability of the observations on all the plots is given in suitable terms (sum of squares) it may legitimately be apportioned to various known causes, leaving a remaining portion ascribable to uncontrolled or unknown causes. The latter will then serve as a basis for the calculation of the error of the experiment. The variance (standard deviation squared) due to any of the known causes or to the uncontrolled or unknown causes may then be found by dividing the sum of squares by the appropriate number of degrees of freedom. The term "degrees of freedom" is here used in the sense of "independent comparisons." With n quantities whose mean is fixed there are in general $n-1$ independent comparisons, or degrees of freedom. The data obtained in this study were analyzed by this method, and the significance of the results obtained was determined by reference to tables given by Fisher [10] for testing the significance of such results.

EXPERIMENTAL RESULTS

ANALYSIS OF YIELD DATA

The analysis of variance of yield, sugar percentage, and apparent purity was made on the assumption of five varieties or treatments to be tested. If the arrangement of the varieties within each replication series, or block, is a random one, it is legitimate to remove the sum of squares attributable to variations between blocks from the total sum of squares, leaving the sum of squares due to variation between plots within blocks from which to calculate the error of the experiment. Ordinarily this will give a lower estimate of the error of the experiment than the total standard deviation. Usually an appreciable portion of the total variance can be removed in this way. The effect of increased replication can be determined easily. The standard error of the mean of several replications would be found by dividing the standard error (standard deviation) of a single plot by $\sqrt{N}$, where N is the number of replication series, or blocks.

The analysis of yield data will be given first. The plot yields from which these analyses were made are given in Table 1. The standard errors between plots within blocks were calculated for 24 different sizes and shapes of plots, considering the entire plot as harvested. With plots 3 or more rows wide, a single row could be removed from each side of the plot to correct for competition between varieties. The standard errors were calculated, therefore, on this basis also for plots 3 or more rows wide and the results compared with those obtained when the border rows in each plot were not removed.

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[9] Fisher, R. A., and Mackenzie, W. A. studies in crop variations. ii. the manurial response of different potato varieties. Jour. Agr. Sci. [England] 13: [311]-320. 1923.

[10] Fisher, R. A. statistical methods for research workers. Ed. 3 rev. and enl., 283 p., illus. Edinburgh and London. 1930.

TABLE 1.—*Yield of beets (pounds) from 600 single-row plots, each 2 rods long, with 22 inches between rows*

Row No.	Yield (pounds) from block No. —										Total yield
	1	2	3	4	5	6	7	8	9	10	
1	45.3	54.0	47.7	50.8	43.3	51.7	47.4	48.1	53.9	54.8	497.0
2	47.8	54.3	47.2	42.6	41.7	46.1	45.3	47.8	47.3	51.4	471.5
3	46.0	49.2	48.5	47.7	46.1	53.4	49.3	47.6	54.8	59.8	502.4
4	47.3	43.8	53.4	59.5	40.8	38.8	47.3	43.6	47.6	45.6	467.7
5	43.7	47.7	50.7	45.1	51.5	49.0	42.6	41.9	45.5	42.7	460.4
6	48.1	48.8	57.7	48.1	44.8	41.8	45.7	48.6	49.3	56.3	489.2
7	45.6	46.0	42.7	42.8	51.5	47.7	48.8	47.1	49.4	49.5	471.1
8	48.7	44.9	46.0	48.3	39.0	44.7	48.5	41.6	38.3	53.0	453.0
9	37.1	48.1	39.4	46.3	48.5	50.3	50.2	50.0	41.9	58.1	469.9
10	41.4	51.2	47.6	46.7	45.2	48.2	48.0	51.1	52.0	59.1	490.5
11	43.3	47.6	52.8	50.5	47.6	47.6	48.4	45.3	53.6	57.3	494.0
12	47.2	51.8	49.6	47.2	48.7	42.8	54.2	55.9	53.3	54.7	505.4
13	52.8	49.7	52.9	52.5	49.4	52.4	54.2	48.2	50.6	51.7	514.4
14	52.5	57.1	50.1	46.2	53.4	48.7	52.1	52.6	63.3	52.6	528.6
15	43.7	43.0	51.0	47.3	56.3	44.0	44.1	43.2	57.8	50.0	480.4
16	61.2	56.3	53.8	54.7	59.1	49.1	55.7	48.0	57.6	51.7	547.2
17	55.7	49.7	55.6	45.9	54.2	46.4	61.1	54.7	56.4	51.2	530.9
18	43.5	48.5	52.6	50.1	51.6	48.1	52.7	48.1	48.0	50.5	493.7
19	59.9	48.7	50.9	48.3	46.3	51.6	52.0	49.6	51.5	55.0	513.8
20	55.1	55.0	54.2	49.9	50.3	58.6	58.5	61.6	62.9	58.1	564.2
21	55.4	49.8	52.8	48.8	49.1	61.7	56.2	55.0	56.5	54.8	540.1
22	41.1	46.0	55.6	51.6	54.1	55.4	49.2	59.6	59.7	58.5	530.8
23	56.0	57.8	52.3	48.8	60.0	56.2	60.0	56.2	56.7	54.5	558.5
24	51.8	53.2	52.0	49.7	53.2	50.8	47.6	52.0	67.7	55.9	533.9
25	51.6	49.6	57.1	49.8	60.0	60.6	57.5	54.5	58.2	45.5	544.4
26	49.8	54.5	47.5	48.1	53.5	49.3	51.0	47.2	60.5	59.2	520.6
27	51.3	52.0	56.3	51.1	54.2	60.6	65.2	56.9	66.5	58.1	572.2
28	49.2	48.5	59.1	60.1	63.8	52.5	54.3	58.6	64.7	61.4	572.2
29	50.1	57.6	62.3	56.1	52.3	63.8	59.8	65.3	54.6	50.5	572.4
30	47.6	48.1	48.2	47.6	54.5	45.3	55.2	52.5	50.7	49.5	499.2
31	39.5	40.0	47.0	52.1	51.8	44.8	53.1	50.7	60.5	54.8	494.3
32	42.2	45.8	50.3	53.3	47.9	57.1	59.7	55.1	58.0	43.3	512.7
33	44.9	52.6	49.5	50.0	44.5	40.7	49.6	60.6	51.1	55.2	498.7
34	41.7	48.9	51.3	55.3	45.7	46.7	44.1	46.2	48.6	54.0	482.5
35	48.8	46.2	49.2	49.9	57.6	48.4	44.8	46.6	51.5	50.4	493.4
36	45.3	52.1	46.7	53.3	53.3	58.0	50.6	56.6	59.2	55.7	530.8
37	51.1	57.6	51.3	62.3	56.1	47.0	52.0	58.7	64.7	54.2	555.0
38	50.7	53.8	50.8	53.1	49.9	52.8	47.6	50.9	55.2	51.2	516.0
39	39.6	43.6	56.3	50.6	51.7	59.6	57.8	52.1	48.3	58.9	518.5
40	48.0	50.8	52.1	50.3	54.3	42.6	46.7	49.3	45.6	56.9	496.6
41	53.8	48.7	49.7	51.9	47.4	48.7	55.2	47.8	50.6	42.5	496.3
42	47.6	46.5	42.5	47.5	46.7	45.5	44.2	45.2	48.0	42.7	456.4
43	52.0	48.8	50.6	46.6	45.5	44.7	48.2	52.8	51.4	50.0	490.6
44	52.0	46.4	45.4	49.1	46.6	46.8	48.7	50.8	55.0	47.7	488.5
45	46.1	51.1	51.8	56.4	50.7	47.8	52.0	54.7	58.0	52.3	520.9
46	46.5	58.8	51.2	53.4	58.9	44.6	50.6	51.3	55.0	53.1	523.4
47	57.3	65.8	52.3	52.6	49.0	52.0	54.8	52.5	55.8	57.5	549.6
48	58.9	65.7	54.7	59.8	57.9	53.7	47.6	52.3	54.2	53.2	558.0
49	42.0	47.8	47.3	58.1	47.4	46.2	46.5	51.7	45.3	46.6	478.9
50	45.7	47.5	42.0	43.7	42.0	44.1	40.1	48.1	43.0	46.5	442.7
51	52.6	53.0	55.0	53.5	52.2	56.2	46.7	56.2	54.9	53.7	534.0
52	41.4	46.1	44.5	49.8	55.7	66.7	63.2	47.5	49.5	46.4	510.8
53	42.3	43.7	46.7	51.7	50.5	52.7	55.5	49.0	41.6	48.6	482.3
54	40.6	47.5	45.3	49.6	55.0	54.1	50.2	51.5	53.4	48.2	495.4
55	52.3	48.6	49.3	57.6	54.0	49.3	46.6	52.0	56.3	50.8	516.8
56	50.9	49.1	51.3	52.3	50.7	54.2	56.8	55.2	53.1	52.5	526.1
57	56.8	55.6	51.7	55.0	52.7	53.9	55.4	55.6	52.5	53.2	542.4
58	56.6	52.1	46.2	53.3	53.2	52.3	54.2	46.7	55.2	44.2	514.0
59	45.5	50.9	54.6	49.9	55.8	56.5	58.4	47.0	55.0	49.0	522.6
60	55.5	47.3	58.8	63.0	61.9	63.1	55.7	60.3	57.1	52.3	575.0
Total	2,918.0	3,024.9	3,043.0	3,067.2	3,070.6	3,048.0	3,098.7	3,087.4	3,218.4	3,136.6	30,712.8

In Table 2 the analysis of variance is given for weight of beets from plots of one row 2 rods long.

The total sum of squares was obtained by squaring the weight of each plot, summing, and subtracting the product of the general total times the general mean. The sum of squares between blocks was obtained by squaring the total weight of each of the 120 blocks, summing, dividing by 5 (the number of elements contributing to each total), and subtracting the same product of the general total times the general mean as used in obtaining the total sum of squares. The sum of squares due to variation within blocks is the difference between the total sum of squares and that portion due to variation between blocks. Since a total of 600 plots was considered, there were 599 (n–1) degrees of freedom attributable to the total sum of squares. There were 120 blocks (of 5 plots each) and consequently 119 degrees of freedom due to blocks; 599 – 119 (or 4 × 120) gives 480 degrees of freedom due to variation between the 5 plots within each of the 120 blocks. The mean square or variance (standard deviation squared) is found by dividing the sum of squares by the appropriate number of degrees of freedom. The standard deviation is the square root of the mean square or variance.

TABLE 2.—*Analysis of variance of weight of beets in single-row plots 2 rods long*

Variation—	Degrees of freedom	Sum of squares	Mean square [a]	Standard deviation	z [b]
Between blocks	119	6,740.3456	56.6416	7.5261	0.4735
Within blocks	480	10,547.7280	21.9744	4.6877	
Total between plots	599	17,288.0736	28.8616	5.3723	--------

[a] Mean square or variance (S. D.^{2}) $= \frac{\text{Sum of squares}}{\text{Degrees of freedom}}$.

[b] z=one-half the difference between the natural logarithms of the two variances, or the difference between the natural logarithms of the standard errors (standard deviations).

The significance of the difference between the variance between blocks and that within blocks was determined by the z test developed by Fisher.[11] The test consists in finding the difference between one-half the natural logarithms of the two variances, or the difference between the natural logarithms of the standard errors (standard deviations), and determining the significance of this difference by reference to tables provided by Fisher.[11] The value of z in these tables is given for two different levels of significance—the 5 per cent point and the 1 per cent point. When z exceeds the 5 per cent point, it is considered that a difference as great as the observed difference will be obtained less than once in 20 trials, from homogeneous material, due to the errors of random sampling. The 5 per cent point is taken as a convenient minimum level of significance. In Table 2, the observed value of z exceeds the 1 per cent point and we conclude that the difference was undoubtedly significant. Since the variance between blocks was significantly greater than the variance within, the elimination of variation between blocks has proved worthwhile. The standard error of a single row 2 rods long was, then, 4.6877 pounds or 9.16 per cent of the mean yield of 51.1880 pounds.

In like manner we may determine the standard error of 3-row plots with the outer row on each side discarded to eliminate any possible differential competition between varieties. A single row is then harvested from each 3-row plot. The analysis of variance is given in Table 3.

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[11] FISHER, R. A. Op. cit.

TABLE 3.—*Analysis of variance of yield of beets in 3-row plots 2 rods long, of which only the central row was harvested*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between blocks	39	1,919.0456	49.2063	7.0147	0.3685
Within blocks	160	3,767.2520	23.5453	4.8523	
Total	199	5,686.2976	28.5744	5.3455	

There are now only 199 degrees of freedom attributable to total variation, since there are two hundred 3-row plots in the entire field of 600 single rows 2 rods long. Each block would require 15 rows. There would be 40 such blocks in the field and these would contribute 39 degrees of freedom, leaving 160 degrees of freedom attributable to variation between plots within blocks.

The observed value of *z* exceeded the 1 per cent point and it can be concluded that the variance between blocks was undoubtedly greater than the variance within blocks. The standard error of a single plot was here 4.8523 pounds or 9.53 per cent of the mean yield of all the central rows in the 3-row plots in the field (50.9215 pounds). The standard error was slightly greater than that found in Table 2 because of the fact that the size of the blocks had been increased threefold, allowing a smaller proportion of the total variability to be attributed to variation between blocks.

The 600 small plots in the field (Table 1) could be combined in various ways to form plots of varying size and shape. On the basis of 5 plots per block, it is possible to consider hypothetical plots 1, 2, 3, 4, 6, and 12 rows wide and 2, 4, 10, and 20 rods long. Plots of 3 or more rows each could be harvested entirely, or the central row or rows alone could be harvested, discarding one border row on each side of the plot. Using these combinations, the entire field is considered each time in studying the variance between plots. In Table 4 is given the standard error in percentage of the mean for these combinations.

TABLE 4.—*Standard errors, in percentage of the mean, of yields of plots varying in size and shape*

Length of plot	Standard deviation of yields (per cent) for plots of indicated width (rows)					
	1	2	3	4	6	12
ENTIRE PLOT HARVESTED						
Rods						
2	9.16	6.77	6.00	6.27	6.33	5.05
4	7.42	5.79	5.38	5.49	5.75	3.48
10	5.79	4.72	4.42	4.01	5.24	3.86
20	4.89	4.14	3.60	4.55	4.90	3.24
CENTRAL ROWS HARVESTED						
2			9.53	8.15	7.34	5.18
4			8.15	5.48	6.48	4.68
10			6.21	4.71	6.08	4.07
20			5.70	4.33	5.77	3.92

The data from entire plots harvested will be considered first. In general the standard error, in percentage of the mean, decreased with increased size of plot, which was to be expected. Increasing the width of the plots from one row to two resulted in a very pronounced reduction in the standard error. Further increase in width of plot resulted in but slightly increased accuracy until the 12-row plots were reached. The standard errors for 6-row plots of varying length were greater than for 4-row or 3-row plots and even greater than for 2-row plots for the 10-rod and 20-rod lengths. Soil heterogeneity on this field apparently was of such a nature that 6-row plots were an undesirable width. That the fertility contour lines of the field were such as to render 6-row plots undesirable will be shown later.

Increasing the length of rows from 2 to 4 rods resulted in greatly reduced standard errors. Further increase in length of plots to 10 rods reduced the error further, but not in proportion to the greater area of land used. Still further increase in length of plot to 20 rods resulted in but slightly reduced standard errors and not at all in proportion to the area of land required. In the 4-row plots the standard error was greater in plots 20 rods long than in plots 10 rods long.

Harvesting only the central row or rows from plots 3, 4, 6, and 12 rows wide gave standard errors greater than when the entire plots were harvested, which was to be expected. In the case of 3-row plots only one-third of the entire plot would be harvested, in the 4-row plots, one-half would be harvested, etc. Increasing the length of plot reduced the standard error in essentially the same ratio as when the entire plot was harvested. Increasing the width of plot and discarding border rows reduced the standard error more rapidly than when the entire plot was harvested, because of the fact that the percentage of the plot actually harvested increased with the use of wider plots. It is to be expected, then, that when border rows are discarded plots of certain widths will prove to be more efficient in their use of the land than plots of other widths, and the most efficient plot will not necessarily be the narrowest.

In Table 5 is given the number of replications needed to reduce the standard error of the mean to 2 per cent. The standard error of the mean of several replications is found by dividing the standard error of a single plot by the square root of N, when N is the number of replications.

TABLE 5.—*Theoretical number of replications needed to reduce the standard error of the mean to 2 per cent*

ENTIRE PLOT HARVESTED						
Length of plot	Number of replications for plots of indicated number of rows					
	1	2	3	4	6	12
Rods						
2	21.0	11.5	9.0	9.8	10.0	6.4
4	13.8	8.4	7.2	7.5	8.3	3.0
10	8.4	5.6	4.9	4.0	6.9	3.7
20	6.0	4.3	3.2	5.2	6.0	2.6
CENTRAL ROWS HARVESTED						
2	--------	--------	22.7	16.6	13.5	6.7
4	--------	--------	16.6	7.5	10.5	5.5
10	--------	--------	9.6	5.5	9.2	4.1
20	--------	--------	8.1	4.7	8.3	3.8

Table 5 brings out in slightly different form the same features apparent from a consideration of Table 4. With a standard error of the mean of 2 per cent the standard error of a difference would be 2 times $\sqrt{2}$, or 2.83 per cent. Adopting twice the standard error of a difference as a convenient minimum level of significance, a difference exceeding 5.66 per cent could be considered significant with the replication numbers given in Table 5. With 4-row plots, of which only the two central rows were harvested, such accuracy could be attained by replicating the 2-rod plots 17 times and the 4-rod plots about 10 times. The theoretical number of replications (7.5) required for the latter size, as given in Table 5, would seem rather too low considering the values found for 4-row plots of other lengths. Ten replications would seem to be more nearly the correct number.

In the analyses of variance leading to the standard errors given in Table 4, the varieties within blocks were considered as side by side. With 6-row and 12-row plots, other arrangements within blocks might be considered also. Three varieties might be grown side by side and the other two end to end with two of the former varieties. Six-row plots would then give a block of the shape shown in Figure 1, A. Twelve-row plots would give a block of the shape shown in Figure 1, B.

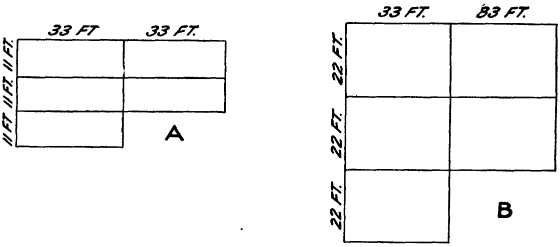

FIGURE 1.—A, Block of beets with five varieties planted in five 6-row plots, each 33 by 11 feet; B, block of beets with five varieties planted in five 12-row plots, each 33 by 22 feet

The arrangement of the plots within these blocks was assumed to be random. For plots 2 rods long, the standard errors in percentage of the mean were found to be 5.46 per cent for the 6-row plots so arranged and 4.87 per cent for the 12-row plots. These were somewhat lower than the standard errors of 6.33 and 5.05 per cent, respectively, found when the five plots per block were side by side. In general it is to be expected that the more compact the block the greater will be the variation removable as variation between blocks, and the lower the standard error within blocks.

In Table 6 is given the efficiency of plots of varying size and shape calculated on the basis of variance per unit area of land. Plots 2 rows wide will require twice as much land as will plots 1 row wide. Plots 3, 4, 6, and 12 rows wide will require a corresponding number of times as much land, respectively, as will single-row plots. The efficiency of plots of different sizes and shapes in their use of the land can then be found by multiplying the variance per plot by the number of single rows 2 rods long which go to make up the plot and expressing the variance of a single row 2 rods long in percentage of these variances. Taking the variance of single rows 2 rods long as a standard, we may determine the efficiency of all other plots in relation to the efficiency of this ultimate unit of size. For example, the variance of 4-row plots 2 rods long (harvested entirely) was 10.3061. Since this is the variance of the mean of four rows, in the 4-row plots, we multiply by 4 and obtain 41.2244 as the variance of a single row in 4-row plots 2 rods long. Dividing the variance of single-row plots 2 rods long, 21.9744 (Table 2) by 41.2244, we find that the 4-row plots were 53.3 per cent as efficient as single-row plots.

TABLE 6.—*Percentage efficiency in use of land of plots varying in size and shape*

Length of plot	Percentage efficiency of plot of indicated width (rows)					
	1	2	3	4	6	12
ENTIRE PLOT HARVESTED						
Rods						
2	100.0	88.0	77.7	53.3	34.9	27.4
4	76.2	62.5	48.2	35.2	21.2	28.8
10	50.0	37.6	28.6	26.1	10.2	9.4
20	35.1	24.5	21.6	10.1	5.8	6.7
CENTRAL ROWS HARVESTED						
2	--------	--------	31.0	31.9	25.9	26.2
4	--------	--------	21.2	35.3	16.6	16.1
10	--------	--------	14.6	19.1	7.5	8.5
20	--------	--------	8.7	11.3	4.2	4.6

Considering the entire plots harvested, the efficiency in use of land is seen to decrease with increased size of plot. While the standard errors, given in Table 4, decreased in general as the size of plot increased, the reduction was not proportional to the increased size of plot and the result was a reduced efficiency of the larger plots.

The most economical size of plot must then be determined from a consideration of the relative cost of planting, cultivating, and harvesting the larger total area needed for large plots, compared with the increased cost of planting and harvesting larger numbers of small plots in order to obtain the same standard error. For example, 4-row plots 2, 4, and 10 rods long utilized the land approximately one-half as efficiently as single-row plots. If the cost of planting, cultivating, and harvesting the 4-row plots did not exceed the cost of planting, growing, and harvesting one-half that area devoted to single-row plots, it would be more economical to use the 4-row width. If the reverse were true, the single-row plots would be more economical. In general, plots of 6 and 12 rows or plots 10 and 20 rods long would not seem economical on this basis. The increased cost due to devoting more land to the larger plots would probably be greater than the slightly increased cost of planting and harvesting slightly larger numbers of smaller plots to obtain the same standard error of the test.

The efficiency of varying sizes and shapes of plots when the border rows were removed was of even greater interest. It is seen that the 4-row plots were the most efficient in use of the land. There would, therefore, be no advantage in using 3-row plots. A greater area of land would need to be devoted to 3-row than to 4-row plots to obtain the same accuracy in the error determinations. Moreover, greater numbers of 3-row plots would have to be planted and harvested. Under average conditions the increased cost of devoting more land to 6-row and 12-row plots would probably not be compensated for completely by the slightly decreased cost of harvesting a smaller number of 6-row and 12-row plots as compared with 4-row plots. The standard errors for 6-row plots were slightly higher than for 4-row plots. A probable explanation of this will be given later. In general, it is to be expected that the standard error per plot will decrease to some extent with increased size. It would seem from these data that when border rows are removed 4 rows would be the preferable width of plot and the length either 2 or 4 rods.

In order to provide a graphic illustration of the effect of soil heterogeneity on yield, the contour map shown in Figure 2 was constructed. The original yield data given in Table 1 were combined to form 6-row plots 2 rods long. The field was then considered as consisting of 100 such plots. Assuming the average yield of each plot to be at the center, the points at which yields were 5, 10, and 15 per cent above the mean and 5 and 10 per cent below the mean were found by interpolation between adjacent plots. The points found in this way for 90, 95, 100, 105, 110, and 115 per cent of the mean yield of all the plots were then joined and the contour map shown in Figure 2 was constructed.

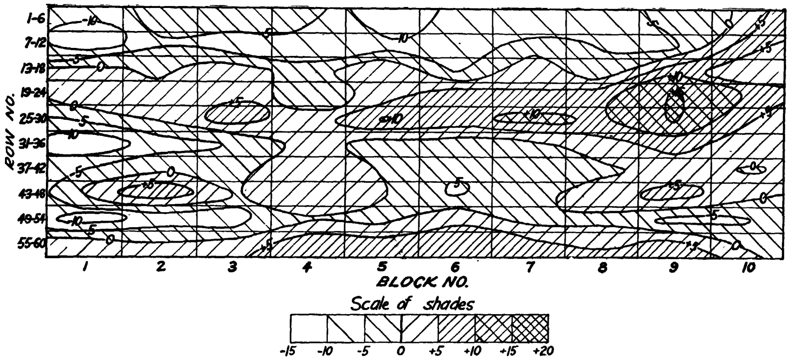

FIGURE 2.—Contour map of weight of beets from one hundred 6-row plots, each 2 rods long; contour lines drawn through points deviating by −10, −5, 0, +5, +10, and +15 per cent from the mean weight

It is quite apparent that the yield varied greatly between different plots in the field. That this heterogeneity was systematic to a considerable extent is also evident. The fertility contour lines were parallel to the rows to a very pronounced degree. The latter fact probably accounts for the high error due to using 6-row plots as compared with other widths, especially when the plots were 4 or more rods long. Other plot widths did not coincide so closely with the inherent soil differences and resulted in lower standard errors. If the rows had been planted at right angles to the direction actually used, the standard errors between plots would have been reduced materially. The direction of these fertility contour lines could not be determined, however, until after harvest.

ANALYSIS OF SUGAR-PERCENTAGE DATA

Since the 4-row plots seemed of greatest interest, particularly when the border rows were removed, the standard errors for sugar percentage and apparent purity were calculated for this width of plot alone. In Table 7 is given the analysis of variance of sugar percentage for 4-row plots 2 rods long with only the central two rows harvested.

There were five plots per block as before. The actual data from which these analyses were made are given in Table 8.

TABLE 7.—*Analysis of variance of sugar percentage from 4-row plots 2 rods long, of which only the central two rows were harvested*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between blocks	29	36.5367	1.2599	1.1225	0.7931
Within blocks	120	30.9512	.2579	.5078	
Total	149	67.4879	.4529	.6730	

TABLE 8.—*Sugar percentage of a 10-beet sample taken from each of 600 single-row plots 2 rods long, with 22 inches between rows*

Row No.	Sugar percentage of 10-beet sample from block No.—										Total
	1	2	3	4	5	6	7	8	9	10	
1	14.60	14.41	13.85	14.24	14.22	13.18	13.66	14.02	14.14	14.58	140.90
2	14.22	14.58	14.29	13.61	13.66	14.20	13.76	14.17	14.07	14.22	140.78
3	14.31	14.52	14.99	14.09	13.76	14.80	13.90	13.81	14.51	14.36	143.05
4	14.58	14.19	14.85	13.68	13.37	13.90	13.97	13.51	14.41	14.10	140.56
5	14.14	13.66	14.40	14.17	13.83	13.20	14.22	14.34	14.36	14.63	140.95
6	14.84	13.87	14.41	13.74	14.41	13.69	13.68	14.26	15.19	14.52	142.61
7	15.38	14.14	13.44	13.95	13.35	13.71	13.20	14.43	15.87	14.60	142.07
8	14.82	14.09	14.38	14.64	14.29	14.27	13.97	14.58	14.24	14.16	143.44
9	15.87	13.37	13.66	13.66	13.98	14.58	13.90	13.51	14.48	15.65	142.66
10	13.74	13.90	14.24	13.76	13.68	13.95	13.71	13.53	14.31	15.04	139.86
11	13.88	13.29	14.87	14.48	13.71	13.27	14.27	14.29	15.50	14.87	142.43
12	14.46	14.31	14.17	14.02	14.16	14.07	14.31	14.14	14.60	15.38	143.62
13	13.81	14.48	13.51	14.82	13.49	14.14	14.05	13.29	14.38	14.90	140.87
14	14.77	14.60	14.34	13.95	14.07	14.00	14.25	14.24	15.02	14.02	143.26
15	14.07	14.26	14.78	13.81	14.35	14.22	13.95	14.36	14.41	13.61	141.82
16	14.76	14.03	13.44	14.22	15.47	13.95	13.90	14.58	14.85	14.92	144.12
17	14.76	13.52	14.00	14.31	14.65	14.43	14.14	14.79	13.71	15.06	143.37
18	13.51	13.41	14.27	13.74	14.80	13.79	14.48	14.55	14.02	14.29	140.86
19	14.31	13.66	14.55	14.00	14.63	14.09	13.78	14.05	14.12	14.58	141.77
20	14.16	15.04	14.17	13.88	13.93	13.97	14.75	14.92	14.51	14.66	143.99
21	13.90	13.44	13.95	14.51	14.58	14.55	14.00	14.51	14.60	14.38	142.42
22	14.73	14.05	14.24	14.94	15.61	14.71	14.35	14.82	14.84	14.84	147.13
23	14.00	13.97	14.43	14.65	13.95	14.24	13.43	14.24	13.92	14.60	141.43
24	14.53	14.25	13.74	14.19	14.77	13.71	15.19	14.16	14.12	14.52	143.18
25	13.64	13.61	15.36	14.09	14.22	14.41	13.83	14.38	14.15	14.82	142.51
26	14.17	13.57	14.21	14.87	14.74	14.45	14.31	14.85	13.73	15.02	143.92
27	14.48	13.15	14.82	14.97	14.12	14.71	14.02	14.70	13.93	15.19	144.09
28	14.20	14.26	14.00	14.21	13.88	14.27	14.12	15.19	15.20	14.22	143.55
29	13.78	13.99	14.12	14.48	14.58	14.09	14.43	14.07	14.70	14.68	142.92
30	13.80	14.52	15.43	14.70	14.32	14.31	14.60	13.83	14.75	15.84	146.10
31	13.90	14.87	15.01	14.82	15.40	14.80	14.00	14.51	14.82	15.41	147.54
32	14.31	14.28	14.24	14.68	14.14	14.77	14.58	14.31	14.68	15.75	145.74
33	14.17	15.00	15.09	14.55	14.09	14.84	15.24	13.88	14.87	15.07	146.80
34	15.28	14.71	15.21	15.24	15.47	14.60	14.40	14.21	15.17	15.30	149.59
35	14.65	14.43	14.58	14.84	14.75	13.90	14.73	14.45	14.82	15.28	146.43
36	14.55	15.17	14.24	14.53	14.16	14.97	14.48	14.58	15.94	14.28	146.90
37	14.07	15.28	15.94	14.48	14.48	15.35	14.79	14.16	14.63	15.90	149.08
38	14.61	14.48	15.45	14.80	14.55	14.84	14.27	14.53	14.51	15.26	147.30
39	15.40	15.17	15.04	14.31	14.67	13.78	14.27	14.24	15.87	15.83	148.58
40	14.46	14.38	14.97	14.38	14.46	14.66	14.07	15.22	15.00	14.14	145.74
41	14.46	14.84	14.63	14.48	14.61	15.04	14.63	15.01	15.50	14.96	148.16
42	13.90	14.29	14.50	14.53	15.04	15.04	15.33	14.31	15.07	14.53	146.54
43	14.00	14.84	14.50	13.95	15.04	14.94	13.92	13.73	14.07	14.35	143.34
44	14.58	14.19	14.48	14.70	14.12	14.16	14.14	14.05	15.31	15.29	145.02
45	14.43	14.80	14.09	14.61	14.48	14.83	14.65	14.92	14.27	16.23	147.31
46	14.80	14.31	14.80	14.38	14.09	14.58	15.45	13.90	14.80	14.91	146.02
47	14.41	14.92	14.21	14.73	14.46	14.93	14.38	14.63	14.99	15.60	147.26
48	14.90	14.43	14.80	14.63	14.80	14.37	14.99	14.55	14.55	15.87	147.89
49	14.35	14.05	15.15	14.80	14.58	14.70	14.48	15.82	14.89	14.85	147.67
50	15.14	14.60	14.68	14.92	14.66	15.06	15.47	15.00	14.76	15.82	150.11
51	15.47	14.63	14.22	13.76	14.82	15.07	14.71	14.92	14.66	15.16	147.42
52	14.92	14.48	14.46	15.01	15.11	15.07	15.19	14.82	15.04	16.18	150.28
53	14.55	14.27	15.10	14.82	15.06	14.80	15.04	14.92	15.19	15.34	149.09
54	14.63	15.14	13.83	14.58	15.09	14.81	14.73	14.87	15.38	15.41	148.47
55	14.63	14.75	14.99	14.48	14.34	15.07	15.01	14.58	15.35	14.84	148.04
56	15.01	15.41	14.24	15.14	14.92	15.09	15.04	14.60	15.26	15.18	149.89
57	13.86	14.99	14.97	14.50	16.27	15.37	15.06	14.17	15.19	16.20	150.58
58	14.14	15.00	14.85	14.31	14.94	14.29	15.28	15.07	14.52	15.49	147.89
59	15.51	14.70	14.80	14.46	14.24	14.89	14.60	14.58	15.24	15.89	148.91
60	15.31	14.53	13.95	14.98	15.36	14.97	14.73	15.31	14.70	16.26	150.10
Total	868.62	861.08	869.93	864.78	868.78	866.45	863.79	865.97	883.69	900.84	8,713.93

The observed value of z exceeds the 1 per cent point, indicating that a significant gain has resulted from eliminating the variability between blocks. The standard error between plots within blocks was 0.5078, or 3.50 per cent of the mean sugar percentage (14.5154), on the basis of a single 10-beet sample per plot.

In like manner the standard error between plots within blocks for similar 4-row plots 4, 10, and 20 rods long, on the basis of a single 10-beet sample per 2 rods of plot, was found to be 0.3971, 0.2356, and 0.2118, respectively. This would indicate that the variability in sugar percentage was reduced considerably by the increased size of sample from the longer plots. The standard error of sugar percentage within plots would be influenced by the size of the sample taken. The standard error of the mean sugar percentage between plots could be reduced by both replication and size of sample per plot. Both must be considered in deducing the total number of beets per plot necessary for sugar determinations and the number of replications needed in order to reduce the error to a given level. A more complete discussion of this has been given previously.[12] An approximation to the sampling error may be obtained from the variance between the two rows sampled in each 4-row plot. Such an analysis of variance is given in Table 9 for plots 2 rods long.

TABLE 9.—*Sampling error of sugar percentage from plots 4 rows wide and 2 rods long, of which only the central two rows were harvested*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between plots within blocks	120	30.9512	0.2579	0.5078	0.2424
Within plots	150	23.8215	.1588	.3985	
Total within blocks	270	54.7727	.2029	.4504	--------

z was greater than the 1 per cent point, indicating that the variance between plots was significantly greater than the variance between the two rows within the plots. In so far as the variance between two samples from adjacent rows within the plots gives the same result as would be obtained by taking two 10-beet samples uniformly over both rows, the results may be taken as a measure of the sampling variance within plots. The variance between rows within plots on this basis could be reduced in direct proportion to the size of sample. The difference between variance between plots and within plots (0.2579 − 0.1588 = 0.0990) would measure the response due to inherent soil differences between plots and could be reduced by increased replication alone. Sixty-two per cent (0.1588 ÷ 0.2579) of the variance between plots, therefore, was due to sampling error. A study was made previously [13] with individual sugar analyses on 10 beets taken from each 2-rod plot from rows 14, 18, 22, 26, 30, 34, 38, 42, 46, and 50. (Table 8.) The present study covered a greater area and was made on bulk analyses of 10 beets instead of 10 individual analyses on as many beets. A comparison of the results might be of interest, however. In the study on individual beets the variance between the

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[12] IMMER, F. R. Op. cit. (Footnote 5.)

[13] IMMER, F. R. Op. cit. (Footnote 5.)

means of 4-row plots (only one row sampled) was 0.2925, the variance within plots (single rows) was 0.2137, and the total variance within blocks 0.2202. The data from the present study (Table 9) compared quite favorably with those results, considering the difference in area covered by the experiment, as well as the other modifications. The variance obtained in the present study from bulk analyses on 10 beets when these were ground up entirely was slightly lower than the average of 10 single beet analyses from borings through the center of the beets. It would seem, then, that the studies on size of sample made previously probably gave a conservative estimate of size of sample needed to reduce the standard error of the mean to a given level.

In Figure 3 is given the sugar-percentage contour map of the plots considered in Figure 2. The contour lines were drawn through the points where the sugar percentages were 96, 98, 100, 102, 104, and 106 per cent of the mean sugar percentage of all the plots. The points used in drawing the lines were found by direct interpolation, as described for the yield contours in Figure 2. The original data are given in Table 8.

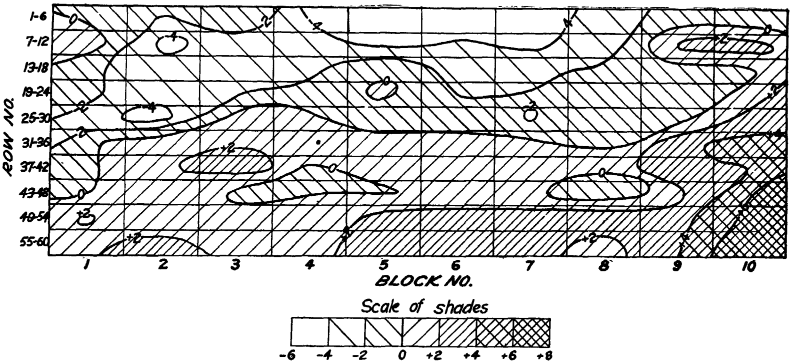

FIGURE 3.—Contour map of sugar percentage of beets from one hundred 6-row plots, each 2 rods long; contour lines drawn through points deviating by −4, −2, 0, +2, +4, and +6 per cent from the mean sugar percentage

These contour lines also ran parallel to the direction of the rows, as found for yield, to a pronounced degree but not to the same extent, especially toward the right-hand side of the field. The sugar-percentage contours did not parallel the yields to an appreciable degree. The only similarity lay in the general tendency for plots giving high yields to have slightly higher sugar percentages as well, and vice versa. The actual regression of sugar percentages on yield and the tendency for soil heterogeneity to affect yield and sugar percentage independently will be given later.

ANALYSIS OF APPARENT-PURITY DATA

The analysis of apparent purity (expressed in per cent) for plots 4 rows wide, with only the central two rows harvested, can be made in a manner identical with that used for sugar percentage. The actual data from which these analyses were made are given in Table 10. Such an analysis of variance for plots 2 rods long is given in Table 11.

TABLE 10.—*Apparent purity percentage of a 10-beet sample taken from each of 600 single-row plots 2 rods long, with 22 inches between rows*

Row No.	Apparent purity percentage of 10-beet sample from block No.—										Total
	1	2	3	4	5	6	7	8	9	10	
1	85.4	84.8	81.0	84.8	83.6	81.9	82.3	85.5	84.2	86.8	840.3
2	84.6	84.8	86.1	82.0	82.8	87.6	83.4	84.8	85.8	84.6	846.5
3	84.7	83.0	85.7	81.4	83.9	86.0	84.2	82.2	85.4	85.5	842.0
4	84.3	83.0	88.4	82.9	81.5	84.8	85.2	80.4	85.8	83.9	840.2
5	86.5	83.8	84.2	84.4	82.3	83.0	87.2	84.8	85.0	86.1	847.3
6	86.3	81.1	85.8	80.8	86.3	86.1	83.9	84.4	88.3	84.9	847.9
7	86.4	84.2	80.5	84.0	81.4	84.6	83.0	85.9	89.2	84.4	843.6
8	84.7	82.4	84.1	84.6	85.1	84.9	85.2	85.8	83.3	82.3	842.4
9	89.7	79.6	80.4	80.8	82.2	85.3	81.8	80.9	85.7	87.4	833.8
10	80.8	82.7	85.8	81.9	82.9	83.0	85.7	79.6	85.2	86.9	834.5
11	81.7	77.7	86.0	84.7	81.1	79.0	84.9	84.6	89.1	87.0	835.8
12	85.6	84.2	83.9	86.5	83.3	85.3	85.2	83.7	85.4	87.9	851.0
13	84.2	86.2	79.9	86.2	82.8	81.7	83.6	80.8	85.1	88.2	838.7
14	85.4	84.4	85.4	83.0	83.8	83.3	86.4	84.3	88.9	81.0	845.9
15	82.8	82.4	88.5	82.2	83.4	84.1	82.5	86.0	85.8	80.1	837.8
16	87.3	84.0	80.5	84.1	86.9	83.0	83.7	84.8	87.9	85.8	848.0
17	86.8	84.0	83.8	83.7	84.7	84.9	84.7	87.5	80.7	86.1	846.9
18	79.9	79.8	84.4	83.8	84.6	82.6	83.2	82.2	81.0	79.8	821.3
19	84.2	83.8	87.1	84.8	88.1	81.9	83.0	86.2	84.6	84.3	848.0
20	82.8	87.4	84.8	85.2	83.4	82.2	85.8	87.8	86.4	86.2	852.0
21	83.7	83.0	84.6	86.4	85.8	85.6	83.3	85.9	84.9	81.2	844.4
22	86.6	84.1	82.3	87.9	90.2	87.0	83.4	85.7	86.8	82.9	856.9
23	83.3	85.7	83.9	84.2	82.5	83.3	79.9	84.3	87.0	84.9	839.0
24	86.5	86.9	83.8	81.6	84.4	84.1	85.8	84.3	85.1	84.9	847.4
25	84.7	81.5	89.3	82.9	87.2	84.8	82.8	85.6	86.3	81.2	846.3
26	83.8	83.8	83.1	85.5	83.3	84.0	85.7	87.4	84.2	87.8	848.6
27	85.7	80.2	86.7	87.5	83.6	86.0	87.1	85.5	87.6	87.3	857.2
28	85.5	83.4	83.3	83.1	83.1	86.5	84.6	85.8	88.9	85.7	849.9
29	85.1	84.3	83.1	83.7	83.8	82.4	84.9	83.8	84.5	85.4	841.0
30	80.7	83.9	86.7	85.5	84.2	84.2	83.9	84.8	85.3	88.5	847.7
31	84.8	86.4	87.3	84.7	87.0	87.6	83.3	86.4	86.7	87.1	861.3
32	84.7	84.0	82.3	84.9	82.7	83.9	84.8	83.7	86.9	87.5	845.4
33	85.4	88.8	84.8	86.1	81.9	86.3	87.1	83.1	87.5	87.1	858.1
34	85.8	87.0	86.9	87.6	86.4	84.9	84.7	83.6	82.9	85.5	855.3
35	86.2	84.9	85.3	85.3	85.8	84.8	86.1	82.6	86.2	86.3	853.5
36	84.1	87.2	79.6	86.5	81.4	85.1	83.2	83.3	89.6	81.6	841.6
37	86.8	85.8	85.2	83.2	85.2	85.8	84.5	81.4	83.6	87.4	848.9
38	86.4	83.7	86.8	86.6	84.1	86.8	84.4	86.5	85.4	86.7	857.4
39	87.0	84.3	83.6	83.7	82.9	80.1	83.9	82.3	89.2	87.5	844.5
40	85.1	85.6	85.5	85.6	85.6	86.2	85.3	87.2	88.2	83.2	857.5
41	84.6	84.3	84.1	86.2	85.9	86.4	84.1	84.8	89.6	83.6	853.6
42	82.7	85.1	81.9	85.5	85.9	86.9	89.1	84.2	88.6	81.6	851.5
43	83.3	85.8	83.8	82.5	85.4	85.5	81.4	79.8	82.8	83.4	833.7
44	84.3	85.0	84.2	85.5	83.1	82.3	84.2	84.6	89.0	87.9	850.1
45	84.9	85.1	85.4	86.4	84.2	87.2	84.2	87.8	84.4	89.7	859.3
46	85.6	85.2	85.6	84.1	82.4	85.3	86.8	85.3	87.1	85.2	852.6
47	81.0	88.8	81.2	84.7	85.1	82.0	81.2	85.1	86.2	87.2	842.5
48	88.2	85.4	86.6	87.1	87.6	86.6	85.7	85.6	85.6	85.3	863.7
49	82.0	85.7	88.6	86.0	84.8	85.0	85.7	88.4	85.6	87.9	859.7
50	86.0	84.4	83.9	86.2	85.7	84.6	86.9	86.7	87.3	89.9	861.6
51	88.4	84.6	85.7	81.4	85.7	88.1	86.0	86.2	86.2	84.7	857.0
52	88.8	85.2	85.6	85.8	85.8	86.6	86.3	86.2	85.9	89.9	866.1
53	84.6	83.9	88.8	86.7	85.1	84.6	87.4	86.7	86.8	89.7	864.3
54	85.1	86.0	84.8	85.8	85.2	84.2	86.7	85.0	85.9	88.1	856.8
55	86.6	86.3	85.7	83.7	87.0	87.6	84.8	86.3	87.7	84.8	860.5
56	86.8	89.1	84.8	86.5	87.8	84.3	86.4	84.9	87.2	88.3	866.1
57	82.5	87.2	88.1	83.8	90.9	85.9	85.1	84.9	88.3	90.0	866.7
58	80.3	87.7	87.9	84.2	85.9	84.1	86.3	88.1	84.4	86.8	855.7
59	85.2	84.5	86.6	89.3	81.8	86.6	83.9	86.8	87.1	88.8	860.6
60	89.0	85.5	83.0	83.7	84.4	87.5	86.6	88.0	84.5	88.8.	861.9
Total	5,095.9	5,072.6	5,086.7	5,079.4	5,074.9	5,085.9	5,082.4	5,090.8	5,169.8	5,150.5	50,988.0

TABLE 11.—*Analysis of variance of apparent purity from 4-row plots 2 rods long, of which only the central rows were harvested*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between blocks	29	302.3527	10.4260	3.2289	0.3941
Within blocks	120	568.7860	4.7399	2.1771	
Total	149	871.1387	5.8466	2.4180	--------

The observed value of z exceeded the 1 per cent point and was undoubtedly significant. The standard error between plots within blocks was 2.1771, or 2.56 per cent of the mean apparent purity (84.9815), on the basis of a single 10-beet sample per plot. This is somewhat lower than the standard error calculated from the sugar percentages (3.50 per cent) on the same plots.

The standard errors between plots within blocks for plots 4, 10, and 20 rods long, on the basis of a 10-beet sample taken from each plot 2 rods long, were found by similar analyses of variance to be 1.6651, 1.0971, and 0.7875 per cent, respectively. These variances decreased markedly with increasing size of sample from the longer plots.

A direct comparison may now be given of the standard errors within blocks for yield, sugar percentage, and apparent purity for 4-row plots 2, 4, 10, and 20 rods long when only the central rows were harvested. The yields were obtained from all the beets harvested, and the sugar and purity percentages on the basis of a 10-beet sample per 2-rod plot. Expressing these in percentage of the mean we obtain the results given in Table 12.

TABLE 12.—*Standard errors, in percentage of the mean, of yield, sugar percentage, and apparent purity for 4-row plots of four lengths, of which only the central rows were harvested*

Length of plot	Standard error of—		
	Yield [a]	Sugar percentage [b]	Apparent purity [b]
Rods	*Per cent*	*Per cent*	*Per cent*
2	8.15	3.50	2.57
4	5.49	2.74	1.96
10	4.71	1.62	1.29
20	4.33	1.46	.93

[a] Calculated from total number of beets harvested on plot.
[b] Calculated on basis of a 10-beet sample per 2-rod plot.

Apparently weight was more variable than either sugar percentage or apparent purity, even when the latter was obtained from a 10-beet sample per plot and the former from the entire plot. The standard errors for sugar percentage and apparent purity were reduced in almost direct proportion to the increased size of sample taken from the longer plots and were not greatly affected by sampling over greater areas.

An approximation to the sampling error for apparent purity may be obtained in the manner suggested for sugar percentage. (Table 9.) Such an analysis of variance is given in Table 13.

TABLE 13.—*Sampling error of apparent purity from 4-row plots 2 rods long, of which only the central rows were harvested*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between plots within blocks	120	568.7860	4.7399	2.1771	0.1507
Within plots	150	525.9600	3.5064	1.8725	
Total within blocks	270	1,094.7460	4.0546	2.0136	

The observed value of *z* (0.1507) exceeds the 5 per cent point (0.1417) but not the 1 per cent point (0.2002). The two variances may, therefore, be considered as probably significantly different.

As an approximation to the sampling variance for apparent purity in plots 2 rods long, we may use 3.5064. This is on the assumption that the variance actually obtained from two 10-beet samples taken from adjacent rows within the plots would be very nearly the variance of two 10-beet samples where each was taken from both rows in a random manner. This variance would then be a measure of the sampling error and could be reduced by increasing the size of sample. The difference between the variance between plots and that within plots (4.7399−3.5064=1.2335) would measure the variance due to soil differences between plots. This latter variance could be reduced only by increased replication. Approximately 3.5064÷4.7399, or 74 per cent of the variance between plots, was due to sampling error. Both variance within plots and variance due to inherent soil differences between plots must be considered in estimating the value of replication and size of sample to reduce the standard error of the mean to a given level. Since the standard error for sugar percentage in percentage of the mean was greater than for apparent purity, it would follow that the standard error for purity would usually be lower than for sugar percentage with the same size of sample.

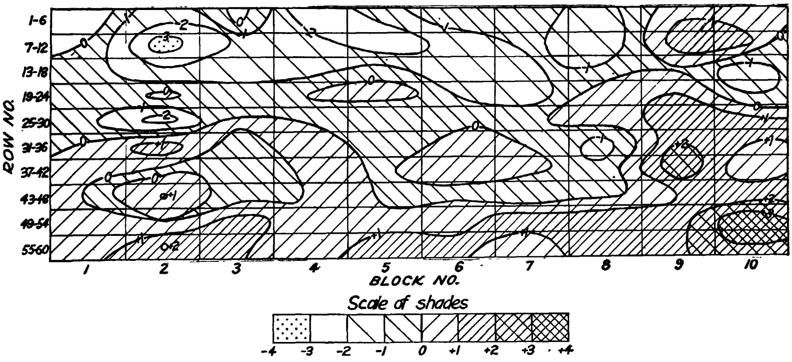

FIGURE 4.—Contour map of apparent purity of beets from one hundred 6-row plots, each 2 rods long; contour lines drawn through points deviating by −3, −2, −1, 0, +1, +2, +3 per cent from the mean apparent purity

In Figure 4 is shown the same type of contour map for apparent purity as was given for weight and sugar percentage. These contour lines were drawn through the points at which the purity was 97, 98, 99, 100, 101, 102, and 103 per cent of the mean.

The contour lines bear a marked similarity to those for sugar percentage but not to those for yield. In general, the areas of high sugar percentage were also high in purity of juice and vice versa. Some exceptions are noted, however. Apparently the field was quite heterogeneous for weight of beets, sugar percentage, and apparent purity. While the two latter bear a distinct relationship to each other, there are marked differences in certain areas.

REGRESSION OF SUGAR PERCENTAGE ON YIELD AND OF APPARENT PURITY ON YIELD AND ON SUGAR PERCENTAGE

Calculations were made for the linear regression coefficients, within plots, of sugar percentage on yield and of apparent purity on yield and on sugar percentage for 4-row plots 2 rods long with only the central rows harvested. The regression coefficient expresses the expected value of the dependent variable on the basis of its relationship to the independent. These may be summarized as follows:

Regression	Regression coefficient (per cent)
Sugar percentage on weight	−0. 020753
Apparent purity on weight	−. 067567
Apparent purity on sugar percentage	3. 334299

The significance of the regression coefficients was tested by the method proposed by Fisher.[14] The regression of sugar percentage on weight of beets was probably significant. The z test showed that the difference between the variations due to linear regression and to departure from regression exceeded the 5 per cent point but not the 1 per cent point. It may be concluded, therefore, that weight probably affected sugar percentage significantly when the relationship was determined within plots. The negative regression of purity on weight was not significant. The observed z value did not exceed the 5 per cent point in the latter case. The regression of apparent purity on sugar percentage was highly significant.

The regression equation may be used to express the estimated value of the dependent variable in relation to the independent variable. This is given in the case of regression of sugar percentage on weight by

$$Z=\bar{z}+b\ (w-\bar{w}),$$

where Z (Zucker) is the estimated sugar percentage and $\bar{z}$ and $\bar{w}$ are the means of sugar percentage and weight, respectively, w is any observed weight, and b is the regression coefficient. Letting P represent apparent purity, the different regression equations may be expressed as follows:

Regression	Regression equation
Sugar percentage on weight	$Z=15.5646-0.020753\ w$
Apparent purity on weight	$P=88.2734-\ .067567\ w$
Apparent purity on sugar percentage	$P=36.4588+3.334299\ z$

In these equations w and z represent any observed value of weight and sugar percentage, respectively, obtained in the experiment.

The intraplot correlation coefficients may be given also, for convenience. The significant coefficients are in italic. If the regression coefficients are significant, it follows that the correlation coefficients must be significant also.

Correlation	Coefficient of correlation
Sugar percentage and weight	*−0. 1746*
Apparent purity and weight	− . 1210
Apparent purity and sugar percentage	*. 7096*

There was little relationship between weight of beets and either sugar percentage or apparent purity. Sugar percentage and apparent purity were highly correlated, as would be expected.

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[14] FISHER, R. A. Op. cit. (Footnote 10.)

The linearity of regression of sugar percentage on weight, apparent purity on weight, and apparent purity on sugar percentage was tested. The regressions were found to be linear.

VARIATION IN SUGAR PERCENTAGE AND IN APPARENT PURITY WHEN THEIR RELATIONSHIP WITH WEIGHT AND SUGAR PERCENTAGE, RESPECTIVELY, IS HELD CONSTANT

It would seem of interest to determine the variability in sugar percentage between plots after correction for regression of sugar percentage or weight within plots. The sum of squares of sugar percentage between plots after such correction would be given by

$$S\{(z-\bar{z})^2-2b\,(w-\bar{w})\,(z-\bar{z})+b^2(w-\bar{w})^2\},$$

where S represents summation, $(z-\bar{z})$ and $(w-\bar{w})$ represent, respectively, any observed deviation of sugar percentage and weight between plots from their mean, and b is the regression coefficient of sugar percentage on weight within plots. Comparing this quantity with the departures of sugar percentage within plots from regression would give an exact test of the significance of variation in sugar percentage after correcting for its relationship with weight. The analysis of variance is shown in Table 14.

TABLE 14.—*Test of variability of sugar percentage between plots apart from its relationship with weight*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between plots, corrected for weight	149	73.9172	0.4961	0.7043	0.5817
Departure from regression	149	23.0954	.1550	.3937	

The observed value of z exceeds the 1 per cent point, and it is concluded that the sugar percentage varied significantly apart from its relationship with weight. In fact, the variance between plots after correction for weight (0.4961) was 9.5 per cent greater than the variance without correction (0.4529). An explanation for this is found in the fact that the regression of sugar percentage on weight within plots was negative (−0.020753), while the sum of products, $S\{(w-\bar{w})\,(z-\bar{z})\}$, between plots was positive (95.9203).

The variation in apparent purity apart from its relationship with sugar percentage should prove of interest, since these two characters are highly correlated. The analysis of variance is shown in Table 15.

TABLE 15.—*Test of variability of apparent purity between plots apart from its relationship with sugar percentage*

Variation	Degrees of freedom	Sum of squares	Mean square	Standard deviation	z
Between plots corrected for sugar percentage	149	397.3653	2.6669	1.6330	0.2100
Departure from regression	149	261.1234	1.7525	1.3238	

The observed z exceeds the 1 per cent point, indicating that apparent purity varied from plot to plot apart from the relationship between apparent purity and sugar percentage within plots. Correction on the basis of the regression of apparent purity on sugar percentage reduced the variance in apparent purity between plots from 5.8465 to 2.6669, or to 46 per cent of the original variance. Therefore 54 per cent of the variation in apparent purity between plots was due to the factors that affected sugar percentage.

These calculations substantiate the conclusion, arrived at previously, that the variation in weight was entirely independent of variation in purity and very nearly independent of variation in sugar percentage. Sugar percentage and apparent purity varied together to an appreciable degree. Slightly more than one-half of the variation in apparent purity was due to factors that affected sugar percentage as well.

DISCUSSION

It would seem, from the data presented here, that fairly narrow plots, either 2 or 4 rods long, would be the most economical size and shape to use for agronomic experiments with sugar beets. Some modifications would need to be made for certain types of experiments. In regions where stands are known to be poor because of unfavorable soil conditions, diseases, or insect pests, larger plots would seem advisable or replication should be increased.

The standard errors obtained from using plots of varying size and shape probably could be considered fairly high estimates of the errors to be expected under average conditions. The contour lines for weight ran parallel to the direction of the rows to a very pronounced degree. This would result in an increased estimate of the standard error between plots. Under average conditions these contours probably would not parallel the direction of the rows to the same degree. The same was true of the sugar-percentage contours and, to a slightly less degree, of the apparent-purity contours. The estimates of the error between plots were probably slightly above that expected under average conditions, assuming environmental conditions similar to those of 1930.

The linear regression of sugar percentage on weight, for individual beet analyses, was found in a previous study[15] to be expressed by the equation

$$b = -0.589375\ (w - \overline{w}),$$

where w was the weight of a single beet. A 1-pound increase in weight would then mean a reduction of 0.59 per cent sugar. In the present study the regression of sugar percentage on weight was expressed by the equation

$$b = -0.020753\ (w - \overline{w}),$$

where w was the weight of a single-row plot 2 rods long. Each such plot contained a maximum of 33 beets. An average increase of 1 pound in weight per beet would mean a decrease of 0.68 per cent sugar (33 times -0.020753), which is in fairly close agreement with the value found for the individuals. The regression of sugar percentage on weight was not entirely linear in the case of the individual beet analyses.[15] The quadratic regression showed that a unit increase

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[15] IMMER, F. R. Op. cit. (Footnote 5.)

in weight did not result in as great a reduction in sugar percentage among the smaller beets as among the larger. One would expect, therefore, that when the sugar percentage was determined from bulk samples of entire beets the larger beets would contribute a greater quantity of juice of lower sugar content and a higher regression coefficient would be obtained. Such was actually the case.

The variability in sugar percentage between plots could not be reduced by means of the regression of sugar percentage on weight. Such regression was negative within plots and positive between plots. The variability of apparent purity between plots could be reduced 54 per cent by holding constant the effect of sugar percentage on purity. It would seem, therefore, that differences in apparent purity between varieties or treatments, apart from the effect of sugar percentage, could be determined with a high degree of accuracy. The general method of determining the variation in sugar percentage and apparent purity apart from their relationship with weight and sugar percentage, respectively, would seem to be extremely valuable in agronomic experiments with sugar beets.

SUMMARY

Studies of size and shape of plot in relation to field experiments with sugar beets have been made, and the relationship determined between weight, sugar percentage, and apparent purity.

Standard errors, expressed in percentage of the mean, decreased in general with increased size of plot. An explanation is offered to account for a greater standard error from 6-row plots than from 3 or 4-row plots, when the entire plot is harvested.

Efficiency in use of land decreased with increased size of plot when the entire plot was harvested. When the border rows of the plots were removed, 4-row plots were most efficient.

Weight of beets was significantly correlated (negatively) with sugar percentage, but not with apparent purity. Sugar percentage was highly correlated (positively) with apparent purity. Intraplot regression and correlation coefficients were given.

Contour maps for weight of roots, sugar percentage, and apparent purity were drawn from data on one hundred 6-row plots 2 rods long.

Sugar percentage varied significantly from plot to plot apart from its relation to weight. Fifty-four per cent of the variability in apparent purity between plots was due to factors that affected sugar percentage as well.

The sampling error was calculated for sugar-percentage and apparent-purity determination for 4-row plots 2 rods long. The manner in which the standard error between plots may be reduced by replication and size of sample has been demonstrated.

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