THE LAW OF DIMINISHING RETURNS IN AGRICULTURE[1]

By P. E. McNall

Professor, Department of Agricultural Economics, Wisconsin Agricultural Experiment Station

INTRODUCTION

A correct understanding of the law of diminishing returns both as to its application, as well as to its statement as a principle, is fundamental in any study designed to measure the effect of increasing units of an input factor upon the output whether this output be in the form of plant growth or animal production.

Immediately following Von Liebig's emphasis of the need by plants of various mineral nutrients necessary for growth, attempts were made to bring under some law the relation existing between added units of any input factor necessary for plant growth and the additional outputs caused by those additional units. Von Liebig developed his law of the minimum, namely, that the productivity of a field is in direct relation to the necessary constituent contained in the soil in the smallest quantity (*8, p. 105*)[2]. In this law he recognized not only the importance of the limiting factor in plant growth but also the necessity of a balanced plant-food supply. The law itself, however, was not correctly stated, and it took the thought and research of four generations of soil chemists to develop a sufficient body of data under carefully worked out control conditions to show (1) that the law as Von Liebig stated it was incorrect and (2) to make a correct statement of the law possible.

DISCUSSION

It was Mitscherlich (*10, 11, 12, 13, 14, 15, 16, 17*) preceded by Mayer (*9*) and Wagner (*20*) who, following years of experimentation, finally restated the law to express the diminishing increments [3] of output with the addition of like units of any input factor. He first experimented with different forms of phosphoric acid fertilizer in pot experiments and showed that as the applications of either of the forms of phosphoric acid were increased the yield also increased, but at a decreasing rate. As the application of monobasic phosphate of lime was increased from 1 to 10 units, for example, the yield increased at the following rate (*19*):

Units applied	Increase in yield per added unit of fertilizer
1. 0	44. 3
2. 5	10. 8
5. 0	7. 2
10. 0	2. 0

[1] Received for publication Oct. 17, 1932, issued August, 1933.

[2] Reference is made by number (italic) to Literature Cited, p. 177.

[3] "Increments" is used to indicate the added output, and "returns", total output.

Journal of Agricultural Research Washington, D.C.

Vol. 47, no. 3 Aug. 1, 1933 Key no. Wis.-57

Later experiments were performed with other fertilizing elements, and these were followed in turn by experiments in which the water and sunlight were varied. The experiment varying the amount of sunlight is interesting. Plants were grown in greenhouse pots shaded by nets varying from 2 to 10 in number for different pots, and the observed growth was compared with full growth in the open. Some of the results are shown in table 1. The increments of growth (or yield) in this instance showed, in general, a decreasing rate in each of the different plants used in the experiment.

TABLE 1.—*Growth increments of plants in greenhouse pots as related to increased intensity of sunlight*

Increase in intensity of sunlight [a]	Increment of growth or yield per unit of added light intensity			
	Oats	Peas	Buckwheat	Lupins
0.025	3.2	1.9	1.6	0.6
.033	3.2	1.4	2	1.2
.044	3.9	1.6	2.2	.9
.063	2.1	.8	1.4	.8
.098	1.7	.7	1.2	.6

[a] In terms of sunlight in the open expressed as 1.00, 0.192 is the least intensity used in the experiment.

It is needless to add to the number of experiments that may be referred to in the United States and in England as well as in Germany to prove the existence of this particular relationship, and it was this type of work which was positively accepted by scientists as proof of the inaccuracy of Von Liebig's statement of the law. It also showed that as any element necessary for plant growth became less (assuming the quantity in the soil to be less than the optimum) the yield was reduced at an increasing rate. Out of this came Mitscherlich's well-known law of the soil.

The next conspicuous development in the theoretical aspects of the law of diminishing increments was to give mathematical expression to the law. Experiments have demonstrated that the cause-result relationships are so certain that they can be shown with mathematical exactitude.[4]

Both Mitscherlich and Spillman, working independently and unknown to each other, showed, through empirical means, that input-output relationships for different plant and animal production assumed a certain form which is expressed by a logarithmic curve. This curve is so constructed that each point on the vertical axis bears the same relation to the maximum vertical rise when expressed in terms of the corresponding point on the horizontal axis as any other point on the axis. If, for example, a unit of any input factor, horizontal axis, results in a yield, vertical axis, of one half the possible maximum which this variant can cause, then the second unit of input will result in the production of one half the remainder; and the addition of a third unit of the input factor will yield one half the remaining possible production which this factor can cause (*1*, *19*).

[4] It should be remembered, however, that results can be predicted with any degree of accuracy only when all the variable factors affecting production or growth are known, and only those are varied whose effects are to be determined.

It is obvious that this curve represents diminishing output per unit of additional input and that exact expression can be given it. In each of these experiments, especially those performed by Mitscherlich, productions were obtained with the variation of a single element of plant growth. Nitrogen, or phosphorus, or sunlight was varied separately, but not two were varied simultaneously. It was in connection with the variation of more than one input factor that Mitscherlich indicated his inability to show and measure results. Baule, however, using some of Mitscherlich's data, deductively arrived at another mathematical expression which when plotted shows a sigmoid or S curve rather than the characteristic logarithmic curve so commonly accepted as the relationship existing between added units of input of any one factor and the corresponding additional outputs.

In working on this problem Baule expressed as 100 the maximum possible yield due to the variation of any single factor (*1*). He then determined the amount of the variable element necessary to produce 50 percent of the maximum, called the Baule unit. If x units of the variable produced 50 percent of the possible maximum, 2x units would produce 50 percent plus one half of 50 percent (remaining production) or 75 percent of the maximum, and 3x units would produce 75 percent plus one half of 25 per cent or 87½ percent, etc. The tabular expressions of the Baule relationships of individual elements of plant growth are shown in table 2.

TABLE 2.—*Baule relationships between individual fertilizing elements favoring plant growth*

Input		Output in terms of maximum output	
Additional	Total	Additional	Total
1	1	½	½
1	2	¼	¾
1	3	⅛	⅞
1	4	1/16	15/16
1	5	1/32	31/32

It was shown that the same relationship would hold for any of the fertilizing elements, so that if the application of nitrogen were varied in one experiment while the application of phosphorus were varied in another it would be possible to make the units of the different kinds of fertilizers of such amounts that the first application in each case would result in an increase in growth equal to 50 percent of the possible maximum growth of the plant; and subsequent applications of the same sized unit would result as indicated in table 2. In other words the curve would be logarithmic.

But what would be the effect of equal doses if a combined unit of nitrogen and phosphorus were applied? It is obvious that the first dose of the combined units of nitrogen and phosphorus would not result in 50 percent plus 50 percent, or 100 percent of the total possible yield. It would be larger than if either had acted alone, but how much larger can be determined only by experimentation. The maximum result of the combined units would again be 100 percent, but this

100 percent seems to bear no known relationship to the "100 percents" representing the maximum possibilities of the respective plant-food elements composing this unit.

If, for example, the greatest possible increase in growth caused by the application of nitrogen to a soil is 40 bushels and the greatest possible increase in yield caused by the application of phosphorus is 30 bushels for the particular soil and climatic conditions in the experiment, it does not necessarily follow that the 100 percent figure for the effect of the combined elements would be 40 plus 30 or 70 bushels. Neither is it logical to expect as small an amount as is received by the application of either of the separate elements alone. Obviously the 100 percent point for the maximum effect of the combined factors would be somewhere between these two points.

Wherever this point is located Baule speaks of it as 100 percent and in order to obtain the relative effect of the simultaneous application of two fertilizing elements he multiplies the proportional yield obtained by one by that of the other. If the first application of nitrogen gives one half of the possible maximum for that element and the first application of phosphorus gives one half of its greatest maximum the effect of the simultaneous application of these two units would be one half times one half, or one fourth of the greatest possible growth, of the combined effects. The second dose of the combined elements would result in three fourths of three fourths or nine sixteenths of the greatest possible growth, and the third dose would result in seven eighths of seven eighths or forty-nine sixty-fourths of the greatest possible growth, of the combined factors. The result of this combination would not be a logarithmic curve but would be a sigmoid or S curve.

The practical significance of this development would be that, whereas the first application (or first unit available for plant growth) of any single fertilizing element or factor of production causes a greater relative growth than any subsequent application of a like unit, the first application of a unit composed of two growth factors would have relatively less effect upon plant growth than would some of the following doses. It necessarily follows from this deductive formulation by Baule that the greater the number of factors of plant growth simultaneously applied, the less will be the relative effect of the first units applied and the greater will be the relative effect of some of the following units (*13*).

It was also brought out by Mitscherlich that the greater the number of variants simultaneously applied, the more delayed will be the point at which the greatest relative effect will take place and the later in the plant growth will the point of diminishing increments occur.

This general relationship is shown graphically in figure 1.

Mitscherlich accepts this as the true relationship between the input and output factors when more than one causal factor comprises the input dose. He shows that it applies to plant growth when the plant development is recorded from a seedling. In his experimentation with sorghum plants in pots under glass the effect of applying equal "quantities of factors of growth" (*13*) composed of several fertilizing elements was shown upon the "accretion" or growth of the plants. Equal growth periods were determined by observing the number of days required for a radish seed to germinate and grow to the point of formation of its second pair of leaves. He hoped in this manner to equalize variations in growth conditions caused by differences in sunlight, humidity, temperature, etc., so that each growth period would have as nearly equal as possible the amounts of those factors of growth which he wished to hold constant. This resulted in a variation of from 11 to 26 days in the time required for different groups of radishes to develop the same amount of growth. Six of the pots were harvested at each of the successive growth periods until some of the plants had grown nearly 6 months. The amount of growth was obtained by determining the grams of dry substance in each "section of growth", or equal growth period.

When these weights are plotted using equal growth periods as the horizontal or variable axis it shows a typical S or sigmoid curve, which is interpreted by Mitscherlich as proving, or at least demonstrating,

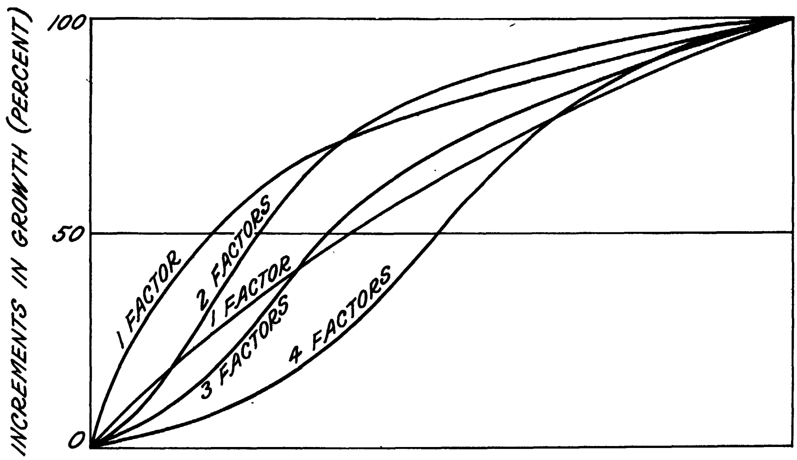

FIGURE 1.—General relationship between the application of various numbers of growth factors and the stage of plant growth at which the point of diminishing returns occurs.

the increasing and decreasing increments of plant growth per unit of input.

Several questions present themselves before this relationship can be accepted as a substitute for the law of diminishing increments or law of physiological relations. It is quite commonly recognized that plant (or even animal) growth is slow in its initial stages of development. Leaf surface for photosynthesis and root area for plant-food absorption are both limited so that only a small amount of plant growth can take place in a given period of time (or growth period). As the plant becomes larger and functional surfaces increase, growth normally becomes more rapid until the plant reaches approximate maturity. At some point in the latter part of the development of the plant the growth is again retarded. This relation of the growth of a plant to the time element has been called the "grand growth curve of plants" and is stated as follows: "Crop growth is accomplished in two principal stages, the first stage being characterized by an increasing rate of growth and the second by a decreasing rate * * *" (*22*, *p. 47*). Growth of this character will take place so long as there are plant-food elements and other factors of growth in sufficient amounts to mature the plant. Growth will be greater in every stage of its development if larger quantities of growth factors are available, but the increasing and decreasing relationships will continue. This particular phenomenon, or characteristic, is not a function of depleted or changing growth factors but is a function of time alone.

Because of this characteristic of plant growth it will be necessary to call into question any results which presume to measure the effect upon crop production of additional applications of some growth factor, as fertilizers, when they are expressed in terms of growth periods. If the fertilizing elements of nitrogen and phosphorus are being applied in increasing amounts, it will be necessary to determine just how much growth is obtained through the addition of each dose of the combined unit. The effect of the first dose upon plant growth cannot be measured until the plant has taken up all of the dose that is biologically possible. The application of the second dose—or two doses to the second plant—must be treated in a manner similar to the first, and so on until maturity is reached. It is probable that when the results of an experiment of this type are plotted it will be found that the first application of nitrogen and phosphorus gives the greatest growth and that each additional dose results in slightly less growth than does the preceding dose.

Experimentation has shown that in some circumstances initial increments of some plant-food element, as nitrogen, when added to a soil deficient in that element may result in no increase in the availability of that element for plant growth—the added amounts being locked in the soil in the form of salts not readily available for plant growth. This results in small increments in plant growth for the initial applications of the fertilizing element. Although experimentation with the simultaneous application of two fertilizing elements has not been recorded where the results are as indicated above, it may be expected that in certain soil and plant nutrient conditions the same relationship may be observed. No generalization should be made from a few instances of this nature, however, as they are more than offset by experiments showing the greatest increments with initial applications. Recognition must also be given to the difference between application and availability.

It should also be remembered that in each of these generalizations it is presumed that what holds for plant growth also holds for crop production. If diminishing increments are characteristic of plant growth under controlled conditions, it is presumed that diminishing increments will characterize crop production of the same plant when grown in larger areas.

It may be desirable to know whether a small dose of fertilizer will have relatively greater effect upon plant growth when applied at the time of seeding, when the plant is 2 inches high, or when the crop is nearly matured, but this is a different question from that of determining the effect of increasing units of fertilizer upon crop growth or production. It is necessary to know how crop yields will be affected with different growth conditions in order that the generalizations may be effectively applied to the industry of farming; and experiments based upon different stages of plant growth which admittedly are characterized by increasing and decreasing increments must be properly interpreted with specific reference to this one characteristic if experiments are to be useful in the general production of the crop.

It should be remembered that the method of combining two or more variables is not limited to one mathematical statement. The only premise from which Baule could have developed the sigmoid curve was by assuming that the initial additions of a combination of input factors result in less production than do subsequent additions; while if still more additions of the input factor are made, production will be at a lessened rate.

Had Baule assumed that the initial application of units [5] of any combination of input factors results in the largest relative increase in production, and that subsequent units result in increases but at a decreasing rate, he could have paralleled the logarithmic curve shown by the use of one variable. Feed as well as fertilizer experiments which are sufficiently complete to throw light on this subject show the decreasing output characteristic.

Many experiments have been set up showing the effect of the functioning of the law of diminishing increments, or the law of the soil, as Mitscherlich expresses it, and the results have been generally accepted. But a few questions should be raised relative to some of the conclusions reached.

It has been shown at different times, for example, that this law functions in the application of water to a crop. Widtsoe and Merrill (*21*) show that when varying amounts of irrigation water are applied to wheat the yield of grain per acre-inch of water decreased consistently from 2.02 bushels per inch for 5 acre-inches of water to 0.77 bushel per inch for 50 acre-inches. A crop like corn which has greater yield possibilities in its present stage of plant development showed the same decline in relation to increasing amounts of water used. The yield of this crop ranged from 6.07 bushels per acre-inch of water with the application of 5 acre-inches, to 1.43 bushels per acre-inch with 55 acre-inches of water. Other crops, without exception, showed the same general trend in yields. Sugar-beet yields were reduced from 0.90 ton per acre-inch of water when 5 acre-inches were applied to 0.41 ton when 50 acre-inches of water were applied.

The authors sum up the presentation of the results of their investigations as follows (*21, p. 114–115*):

> There is a great uniformity among the results obtained when different quantities of irrigation water are applied to the representative crops studied in these experiments. In practically every case, increasing the quantity of water increased the total yield of the crop, but in a smaller ratio as the maximum irrigation was approached. The more water that was used, the smaller was the return per acre-inch of water. So large was this decrease of the acre-inch equivalent that a serious waste of water is to be inferred. As farmers become better acquainted with this truth the problem of the best quantity of water to use must ever confront them with more and more force. Shall the irrigation farmer, having money invested in both land and water, attempt to produce the largest yield per acre or the largest yield per acre inch? Is there some point at which acre-yield and acre-inch-yield meet to produce the largest profits?

In this way the problem is thrown open to solution, and the discussion of the law of diminishing increments as a physical principle becomes identified with the economic concept of diminishing returns. How much land should a farmer use with any given quantity of water and with certain price relationships?

[5] "Application" is used here to indicate availability.

Willcox (*22*), in using this same series of data concludes that there is but one way to move in the utilization of land and that "is toward extensive farming." In proof of this contention he uses the yields of wheat obtained when a given amount of water is applied (30 acre-inches in this case) to 1, 2, 3, 4, and 6 acres of land, the total yields being 47, 91, 130, 166, and 226 bushels from the respective acreages. From this he formulates a law of "increasing increments of yield (variable space)", which is the eighth of his laws of plant growth. "When the amounts of the material factors of plant growth are held constant while the areal space is increased, the yield of the crop is increased at an increasing rate" (*22, p. 46*).[6]

The discussion of this law is not very extensive; but if Willcox means that the aggregate production is increased as more units of a variable factor are applied to a fixed factor, it is self-evident and needs no comment. It matters not whether the product is expressed as a series of totals, or in terms of some fixed factor, as the acre-inches of water in this instance, because whenever an increasing total is expressed in terms of a fixed factor the series must show increasing ratios. If, as the writer believes, he is stating the law of increasing increments it is to be questioned either whether he has correctly interpreted the data or whether the conclusion concerning the application of the law to agriculture is correct.

If the production previously mentioned is expressed as yields per acre of land used it will show 47, 45, 43, 42, and 39 bushels per acre as the land area is increased from 1 to 6 acres. This is a typical illustration of decreasing average returns. If, on the other hand, each increment in production is expressed in terms of the added acreage the following series results, as shown in table 3.

TABLE 3.—*Relation between added acreage and production increments on irrigated land, the amount of water used remaining constant*

Total		Increase in—	
Acres	Yield	Acres	Yield per added acre
	Bushels		*Bushels*
1	47	1	47
2	91	1	44
3	130	1	39
4	166	1	36
6	226	2	30

It is here seen that as the land area is increased—the amount of water used remaining constant—production increases but at a decreasing rate per additional unit of input, or acres.[7] This is another illustration of decreasing returns, rather than increasing returns as stated by Willcox.

[6] The graphic presentation used by Willcox to show increasing returns is in error in that he uses acre-inches of water as the horizontal axis (causal or variable factor) rather than the increase in land surface, in acres. Had he plotted the aggregate yields against acres as this descriptive material suggests, he would have observed the diminishing returns characteristic.

[7] It is conceivable that increasing increments will be in evidence if the amount of irrigation water applied is increased from 1 acre-inch, where no crop production will take place, to several acre-inches, where some crop production is obtained. As production increases from zero to something, the rate of increase is infinite and increasing increments must of necessity be evident. It is not so easily conceivable, however, that, after enough water is applied to obtain some production, further additions of equal amounts of water will result in more than proportionate or even proportionate increases in yield.

In making an application of this law to agriculture Willcox (*22, p. 46*) states:

> The law of diminishing increments of yield is characteristic of extensive farming where land is abundant and the other growth factors, as water, fertilizer, labor, etc., are relatively scarce.

Quite the contrary is true.

Where land is relatively free much more land (per unit of other factors) will be used, but even in this instance extra land will show diminishing rather than increasing increments or returns. It would seem that Willcox has confused total returns with returns per unit of the variable and that he does not recognize the implications of the principle of the conservation of the scarce factor.

Another possible interpretation which Willcox may have had in mind is that as the acre-inches of water become less the yield will become increasingly greater per acre-inch of water. The construction of his chart would indicate this possible interpretation as would also his use of the words "space" and "area". It should be remembered, however, that the aggregate amount of water is held constant according to his premise, and that the varying factor is the area over which the 30 acre-inches of water is applied. The variant is not acre-inches of water but acres of land. When 30 acre-inches of water are applied to 2 acres of land instead of 1 the amount of water applied per acre must be one half as much as when applied to the 1 acre, and when the same amount of water is applied to 4 acres it must of necessity follow that but 7½ inches per acre can be used. If the study is for the purpose of determining the effect of different amounts of water on crop production, the land area should be held constant, as was done by Widtsoe and Merrill in their experimentation.

When an irrigation farmer rents land and water rights he, either thinkingly or not, has determined on a working proportion of land to water. Let us assume that, after farming a few years, he finds his yields of wheat, with the applications of the different amounts of water, to be the same as shown in table 3. He also finds that he is limited to a specific quantity of water during the year. Will he use a few acres of land and apply the water at the rate of, say, 30 acre-inches per acre or will he use six times as many acres and apply 5 acre-inches of water per acre?

This question cannot be answered without knowing the rental charge (or cost) per acre of the land, the cost per acre-foot of water, and the price of the crop. The higher the land rent in proportion to the water cost the fewer the acres of land that will be used. If the farmer had to pay $40 an acre for the rent of land and $2.50 per acre-foot for water, a unit consisting of 1 acre of land and 30 acre-inches of water would cost him $46.25. If wheat sold at $1 per bushel he would receive $47, or a net of $0.75 above rent and water cost for each unit consisting of 30 acre-inches of water and 1 acre of land. If, on the other hand, he applied the same amount of water to a unit consisting of 6 acres of land the cost of the unit in this case would be $246.25 while the returns would be but $226, or $19.75 less than rent and water cost for each unit of land and water. This may be an absurd spread of constant and variable costs, but it helps to illustrate the fact that at times it may pay the irrigation farmer to intensify his operations. If the rental charges were but $10 an acre and other costs and prices were as given above, it would doubtlessly profit the farmer to practice the more extensive type of production. In each instance production is accomplished by decreasing increments per unit of the variable; yet because of the change in relative scarcity (costliness) of one factor the farmer is forced to change the intensity of his farming operations. In no instance is there any indication of increasing increments.

Incidentally if the law of increasing increments of yields as expressed by Willcox does not hold, then his ninth law or "concentration law of growth factors" may well be called into question. This law states that: "In a normal environment the effectiveness of a given quantity of an external factor of crop growth is *inversely proportional* to its concentration per unit of space."

If one unit of fertilizer produces X units of product then, according to the writer's interpretation of this law, 2 units of fertilizer applied to the same area would produce $\frac{X}{2}$ units of the product per unit of fertilizer, or a total of $2 \times \frac{X}{2}$, or X units of product, and the application of 3 units of fertilizer would result in $\frac{X}{3}$ units of product per unit of fertilizer or a total of $3 \times \frac{X}{3}$, or X units. In other words, the greater the application of fertilizer per unit of land the less would be its effect per unit of fertilizer on crop growth. This is obviously in error and cannot be what the author had in mind. If he meant to imply that the influence of each added unit of fertilizer is relatively less than the effect of the preceding unit when applied to the same square foot or acre, this would appear to be a correct statement. It is a restatement of his seventh law, of diminishing increments of yield, however, and should not be set up as a new law.

Another restatement of the law of diminishing increments is found in the eleventh law, action law of growth factors, wherein Willcox (*22, p. 55*) accepts the definite logarithmic formulation of the law as developed by Spillman and Mitscherlich. It is stated as follows:

> When a variable positive growth factor is increased by successive increments the increase in the yield of the crop due to any increment will be proportional to the difference between the total yield produced by the preceding increments and the maximum possible yield * * *.

A third interpretation which may be more in line with the discussion of the author is that, as a given quantity of a growth factor is spread over larger areas, it becomes less concentrated, and the total yield becomes greater per unit of this growth factor which is held constant. This, of course, is true but it is not the statement of a new law. Rather it is a restatement of the truism that the greater the total yield the greater the average yield per unit of any fixed factor.

This principle has also been applied to problems of digestion and metabolism as they affect livestock production. In the different experiments which the writer has reviewed the principle of diminishing increments applies. Planes of feeding work with different classes of livestock, including poultry (*2*), beef (*5, 6, 7*), and dairy production (*3, 4, 18*), all show the general tendency of additional outputs to decrease with added units of any input factor. The authors have not usually set up their data to show this relationship; but wherever experiments are made with this principle in mind, this study should result not only in pointing the way to greater accuracy in research technic but the material itself should also stand up better under the scrutiny of logical interrogation.

LITERATURE CITED

(1) Baule, B.
1918. ZU MITSCHERLICHS GESETZ DER PHYSIOLOGISCHEN BEZIEHUNGEN. Landw. Jahrb. 51: [363]–385, illus.

(2) Buss, W. J.
1915. RATIONS FOR GROWING AND FATTENING ROASTERS AND CAPONS. Ohio Agr. Expt. Sta. Bul. 284, p. 155–172, illus.

(3) Ezekiel, M.
1926. FACTORS AFFECTING FARMERS' EARNINGS IN SOUTHEASTERN PENNSYLVANIA. U.S. Dept. Agr. Bul. 1400, 64 p., illus.

(4) Ezekiel, M. J. B., McNall, P. E., and Morrison, F. B.
1927. PRACTICES RESPONSIBLE FOR VARIATIONS IN PHYSICAL REQUIREMENTS AND ECONOMIC COSTS OF MILK PRODUCTION ON WISCONSIN DAIRY FARMS. Wis. Agr. Expt. Sta. Research Bul. 79, 55 p., illus.

(5) Forbes, E. B., Braman, W. W., and Kriss, M., with the collaboration of Jeffries, C. D., Swift, R. W., French, R. B., Miller, R. C., and Smythe, C. V.
1928. THE ENERGY METABOLISM OF CATTLE IN RELATION TO THE PLANE OF NUTRITION. Jour. Agr. Research 37: 253–300, illus.

(6). ——— Braman, W. W., and Kriss, M., with the collaboration of Swift, R. W., French, R. B., Smythe, C. V., Williams, P. S., and Williams, H. H.
1930. FURTHER STUDIES OF THE ENERGY METABOLISM OF CATTLE IN RELATION TO THE PLANE OF NUTRITION. Jour. Agr. Research 40: 37–78, illus.

(7) ——— Fries, J. A., Braman, W. W., and Kriss, M.
1926. THE RELATIVE UTILIZATION OF FEED ENERGY FOR MAINTENANCE, BODY INCREASE, AND MILK PRODUCTION OF CATTLE. Jour. Agr. Research 33: 483–492.

(8) Liebig, J. von
1855. DIE GRUNDSATZE DER AGRICULTUR-CHEMIE, MIT RÜCKSICHT AUF DIE IN ENGLAND ANGESTELLTEN UNTERSUCHUNGEN. 152 p. Braunschweig.

(9) Mayer, A.
1869. DAS DÜNGERKAPITAL UND DER RAUBBAU. 73 p. Heidelberg.

(10) Mitscherlich, E. A.
1912. ZUR METHODIK DER FELDDÜNGUNGS- UND DER SORTENANBAUVERSUCHE. Landw. Jahrb. 42: [413]–421.

(11) ———
1912. ZUM GESETZ DES MINIMUM. Landw. Jahrb. 42: [423]–424.

(12) ———
1912. DAS WASSER ALS VEGETATIONSFAKTOR. Landw. Jahrb. 42: [701]–717, illus.

(13) ———
1919. DAS GESETZ DAS PFLANZENWACHSTUMS. Landw. Jahrb. 53: [167]–182, illus.

(14) ———
1919. EIN BEITRAG ZUR STANDRAUMWEITE UNSERER LANDWIRTSCHAFTLICHEN KULTURPFLANZEN IN GEFÄSSEN UND IM FREIEN LANDE, BEI REINSAAT UND MANGSAAT. Landw. Jahrb. 53: [341]–360.

(15) ——— and Floess, R.
1912. ÜBER DEN EINFLUSS VERSCHIEDENER VEGETATIONSFAKTOREN AUF DIE HÖHE DES PFLANZENERTRAGES UND ÜBER DIE GEGENSEITIGEN BEZIEHUNGEN DER BODENKUNDLICHEN VEGETATIONSFAKTOREN. Landw. Jahrb. 43: [649]–668, illus.

(16) (10) MITSCHERLICH, E. A., in cooperation with SAUCKEN, S. VON, and IFFLAND, F.
1919. VEGETATIONSVERSUCHE MIT VERSCHIEDENEN KALIDÜNGESALZEN UND ZUR PHOSPHORSÄURE-KALK-DÜNGUNG. Landw. Jahrb. 53: [501]-514, illus.

(17) ——— and SIMMERMACHER, W.
1912. ZUR DÜNGEMITTELANALYSE. Landw. Jahrb. 43: [405]-435.

(18) ROSS, H. A., HALL, H. F., and RHODE, C. S.
1923. THE FEED COST OF MILK AND FAT PRODUCTION AS RELATED TO YIELDS. Ill. Agr. Expt. Sta. Bul. 244, p. 553-573, illus.

(19) SPILLMAN, W. J., and LANG, E.
1924. THE LAW OF DIMINISHING RETURNS; PART ONE: THE LAW OF THE DIMINISHING INCREMENT . . . PART TWO: THE LAW OF THE SOIL . . . 178 p., illus. Yonkers-on-Hudson and Chicago.

(20) WAGNER, P., with the cooperation of ROHN, W., PRINZ, H., WETZKE, T., MYER, C., and LAATSCH, L.
1883. BEITRÄGE ZUR AUSBILDUNG DER DÜNGUNGSLEHRE. Landw. Jahrb. 12: [583]-748, illus.

(21) WIDTSOE, J. A., and MERRILL, L. A.
1912. THE YIELDS OF CROPS WITH DIFFERENT QUANTITIES OF IRRIGATION WATER. Utah Agr. Expt. Sta. Bul. 117, p. [69]-119, illus.

(22) WILLCOX, O. W.
1930. PRINCIPLES OF AGROBIOLOGY; OR, THE LAWS OF PLANT GROWTH IN RELATION TO CROP PRODUCTION. 96 p., illus. New York.